B.Sc Part I (Hons) Paper I

Descartes's Rule of Signs

Establishes Descarate's rule of signs

Statement: If $f(x)=0$, then

(i) The number of positive roots cannot exceed the number of changes of sign in $f(x)$

(ii) The number of negative roots cannot exceed the number of changes of sign in $f(-x)$.

Proof — (1) For positive Roots:

Case I – When polynomial equation cannot have more positive roots than there are changes of signs from + to − and from − to +, in its terms of the corresponding expression

This can be establishes by the following example

$$3x^5-7x^4+3x^3+2x^2-8x-1=0$$

Thes signs of the polynomial are

$$+ \quad - \quad + \quad + \quad - \quad -$$

1 2 3

Here three changes of signs be seen without any loss of generality, let us suppose that the sign of the polynomial expression are

$$+-+--++---+$$

clearly the given polynomial has six changes of Signs.

Now, we multiply the polynomial by a binomial $x-a$ corresponding to a positive root a.

The signs of the binomial $x-a$ are $+-$

Here multiplication can be exhibited as follows:

$$+-+--++---+$$
$$+-$$
$$+-+--++---+$$
$$-+-++--+++-$$
$$+-\mp-+-\pm-\mp\mp+-$$

Now, we are going to show that the [illegible] polynomial has at least one more changes of sign than the original number of changes of signs.

Here we observe 4 ambiguous signs (viz [illegible]) in the resulting polynomial and so it will be $2^4=16$ combinations of these ambiguous sings.

So in this case, the signs of the resulting polynomial are

$$+ - + - - + + - - - + -$$

This gives seven changes of signs. one more than the number of changes of signs in the original.

Case II : In this case the number of changes of signs may be even greater than seven. For this take the first ambiguous sign with a + sign second with a − sign, third with a + sign and fourth sign with a − sign. We shall, then get

$$+ - + - + + - - + - + -$$

This gives nine changes of sings. so in every case there is at least ~~are~~ one variation of sign [illegible]

Thus when the complete product is formed by successively mulplying the polynomial by $x-\alpha$, $x-\beta$, $x-\gamma$ etc, the resulting polynomial will have at least as many changes of signs as it has +ve roots.

(II) Negative Roots :- We know that negative roots of $f(x)=0$ are the positive roots of $f(-x)=0$. So from Descarte's rule of sign for positive roots that the number of positive roots of $f(-x)=0$, cannot exceed the number of changes of signs in the terms of the polynomial $f(-x)$

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